Is Arbitrage Socially Beneficial?*

E. Glen Weyl†

October, 2007

Abstract

Economists often associate informational efficiency of prices with allocative (Pareto) efficiency. When arbitrageurs connect segmented markets to correct a misallocation of risk, this assumption is sound. However, when opportunities for arbitrage or financial innovation arise from mispricings caused by the presence of confused investors or other distortions, such market-making is harmful. Rather than arbitrage facilitating the flow of risk to those who can most efficiently bear it, this harmful arbitrage allocates risk to those who least understand it. The beneficial effects of efficient pricing on real investment decisions mitigates the harms caused by arbitrage, but also limits its benefits by providing substitute insurance. Even if arbitrage is not strictly harmful, it may be oversupplied, especially given that it typically employs some of the most talented workers.

EXTREMELY PRELIMINARY AND INCOMPLETE.

*This paper was written while I was visiting at the Becker Center on Chicago Price Theory at the University of Chicago and I am grateful to the Center for their support of my research. Discussion with David Laibson and Josh Schwartzstein were helpful in focusing my thinking on this topic. I am grateful to my advisers, Hyun Shin and José Scheinkman, for their support and guidance on this and other projects.

†Bendheim Center for Finance, Department of Economics, Princeton University, 26 Prospect Avenue, Princeton, NJ 08540: eweyl@princeton.edu.

1 Introduction

Economists often assume arbitrage, financial innovation and other "market-making" activities are socially beneficial, at least when they succeed. It is argued that they both promote the efficient allocations of risk (see, for example, Shiller (1993)) and help make investment more rational (see, for example, Baker et al. (2002)). While a large literature has emerged about the limits on the ability of arbitrage to bring prices to fundamentals, the implicit assumption in this literature is that it would be good if this arbitrage succeeded. The purpose of this paper is question that proposition.

I consider the simplest possible model of market-making "arbitrage", which I henceforth refer to as arbitrage, though it may also include financial innovation or other forms of market-completion. Two markets for a single risky asset are segmented (physically, institutionally, informationally or temporally) and unable to trade with one another without the help of an arbitrageur. In the classic case, when arbitrage is beneficial, the initial distribution of the asset between the two markets is uneven (undiversified) and therefore Pareto-inefficient. The entrance of an arbitrageur helps smooth risk, making all parties better off. This is classic gains from trade: it is the same as saying that the explorers who traded British luxuries for Indian spices benefited both countries.

On the other hand, suppose that the initial allocation of the asset is smooth (Pareto-efficient), but investors in one market believe the asset to be worth more, and consumers in the other market believe it to be worth less, than it really is. These mistaken beliefs can either be taken literally (investors are "behavioral") or as a metaphor for regulator or agency distortions in the relationship between the investor and his or her investment manager. Then arbitrage actually creates additional risk, moving away from (objective) Pareto-efficiency as investors bet against one another on the basis of mistaken beliefs. Again, the objective measure of social welfare here used to judge can either be taken literally (alla Campbell (2006)) as a paternalistic social planner's preference or simply as a positive argument that arbitrage need not "smooth" risk across investors. In this case, the arbitrageur is rather like a tobacco manufacturer who sells cigarettes to a consumer who genuinely believes cigarettes are good for her health.

It is not clear which form of arbitrage is more common, but economists[1] who argue for the

[1] Such as DeLong et al. (1990).

importance of consumer confusion in the emergence of arbitrage opportunities should be aware of the normative implications of this view. At a time when the financial industry constitutes an increasingly large fraction of aggregate economic activity[2] and employs a large portion[3] of the most wealthy and educated individuals[4], these issues seem particularly pertinent. Furthermore, recent turmoil in credit markets have raised concerns in the popular press[5] that financial innovation and arbitrage may be helping risk to flow to those who least understand it, rather than those most able to bear it. Below I develop a simple formalization of this argument.

After this introduction, the paper is divided into five sections. Section 2 lays out the basic argument and provides a simple formula for mapping an economist's beliefs about how common the beneficial and harmful arbitrage situations are to beliefs about whether arbitrage is on average beneficial. Section 3 discusses different interpretations of the market segmentation assumed in the model and argues that the model may be revelent even for economists who reject the idea that investors are confused or that their decisions should be judged by a paternalistic social planner. Section 4 extends the model to include the possibility that asset prices affect real decisions. The importance of asset prices for capital allocation mitigate the net harm from arbitrage, as such activity stops resources being wasted responding to mispricings. However, this possibility for substitution also reduces the benefits from arbitrage in the beneficial arbitrage setting, as consumers in each market are able to partially insure themselves. Section 5 extends the model by including the costs of producing arbitrage (discovering arbitrage opportunities) in terms of the lost time of the talented workers often employed in the arbitrage sector. Even when it is not optimal to eliminate arbitrage, it may be optimal to tax it, as it is oversupplied, wasting valuable labor resources. Section 6 concludes with some directions for future research.

[2]NEED CITATION FOR THIS

[3]More than twice of the top 1% of income earners as all other executive positions, according to Kaplan and Rauh (2006).

[4]See Murphy et al. (1991) for a discussion of the macro-economic importance of the careers pursued by highly talented individuals.

[5]For a nice informal summary of the role of financial innovation in the recent credit crisis expressing this concern, see *The Economist* (2007).

2 Basic model

Consider an extremely simple economy with two periods. Two (representative) consumers have identical utility $U(c_1, c_2) = c_1 - e^{-rc_2}$, where $r > 0$ measures the consumers' risk aversion. First period consumption is as simple numeraire that makes calculations easier and is not meant to reflect a lack of consumer risk aversion. To further simplify calculations, consumption is not constrained to be positive. Both consumers are (subjective) expected utility maximizers. In period 1 each consumer is endowed with zero units of the consumption good. In period 2, all consumption derives from a risky asset which yields a real dividend X in period 2 which is distributed (under the objective measure) $N(0,1)$. Consumers cannot save or otherwise transfer income between the two periods, except through purchases or short sales of the asset. The two consumers cannot trade directly with one another, but there is an arbitrageur (with the same preferences as the two consumers) who (after entering the market) can buy or sell the risky asset at (internal) market prices in both consumers' markets.

I consider two cases.

2.1 Beneficial arbitrage

First, suppose that consumer 1 is endowed with one unit of the risky asset and that consumer 2 is endowed with negative one unit of the risky asset. Furthermore suppose that both consumers are rational, in the sense that they (correctly) forecast that the risky asset will be distributed $N(0,1)$. Then the price of the asset in the market inhabited by consumer 1 (in the absence of an arbitrageur) is given by the standard asset pricing formula:

$$E\Big[re^{-rX}X\Big] = \frac{r}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-rx}\cdot e^{-\frac{x^2}{2}}xdx = \frac{re^{\frac{r^2}{2}}}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{(x+r)^2}{2}}xdx = -r^2e^{\frac{r^2}{2}}$$

The price of the asset in consumer 2's market is (by the opposite reasoning) $r^2e^{\frac{r^2}{2}}$. Thus the arbitrageur will observe an opportunity for pure arbitrage by buying the asset cheap in consumer 1's market and selling it dear to consumer 2. Of course prices will change as the arbitrageur carries out her trade. Market microstructure will determine the total amount of the potential surplus that can be realized, as well as the fraction of this total captured by the arbitrageur. To abstract from these microstructure issues, I assume all surplus available from the trade is realized (that is, a Pareto optimal allocation of 0 units of each asset in each market is achieved) and that of this total surplus from the trade the arbitrageur captures a fraction α. The second-period welfare of the both of the consumers in the Pareto optimal allocation, where both consumers hold 0 of the asset, is clearly -1 and in the initial allocation is $-e^{\frac{r^2}{2}}$. Thus the total surplus gained by the arbitrage is $2\left(e^{\frac{r^2}{2}}-1\right)$ and the profit earned by the arbitrageur is $2\alpha\left(e^{\frac{r^2}{2}}-1\right)$. If one additionally assumes that the profits of the arbitrageur are paid equally by the two parties then each consumer makes a transfer of $\alpha\left(e^{\frac{r^2}{2}}-1\right)$ to the arbitrageur. Note that clearly as long as $\alpha < 1$ both consumers are strictly better off. Thus in this setting, arbitrage is clearly beneficial both to the arbitrageur and both consumers (and therefore to society). This is a classic example of gains from trade.

2.2 Harmful arbitrage

Now suppose, in contrast to the first case, that the initial allocation is Pareto optimal (under the objective measure) with 0 units of the asset owned by each consumer. However, suppose that consumer 1 mistakenly believes that the asset's dividend is distributed $N\left(-r^2,1\right)$ and that consumer 2 mistakenly believes that the asset's dividend is distributed $N\left(r^2,1\right)$. As discussed below, this assumption of "mistaken beliefs" may be taken literally or seen as a reduced-form proxy for a number of possible distortions. By the same reasoning as above, the price of the asset in consumer 1's market will be $-r^3$ and in consumer 2's market will be r^3. Again, the arbitrageur will observe an arbitrage opportunity from consumer 1's to consumer 2's market. To determine the (subjective) Pareto optimal allocation, note that if consumer 1 is endowed with y_1 units of the asset, then by the standard formula for CARA-Gaussian decision making her (subjective) utility is a positive monotonic function of:

$$-r^2 y_1 - \frac{r^2 y_1^2}{2}$$

Thus if it were costless to endow consumer 1 with the asset it would be (subjectively) optimal to endow her with -1 unit of the good. By the same reasoning it is (subjectively optimal to endow consumer 2 with 1 unit of the good. Given that this unconstrained optimal endowment is feasible by transferring one unit of the asset from 1 to 2, this is clearly the unique (subjective) Pareto-optimal allocation. (Subjective) Welfare in each market in period 2 is initially -1 and after the arbitrage is $-e^{-\frac{r^2}{2}}$. Thus, if the arbitrageur captures a fraction α of the subjective welfare gain and this is (as above) paid symmetrically by the two sides, each consumer will make a transfer of $\alpha\big(1-e^{-\frac{r^2}{2}}\big)$ to the arbitrageur.

However, under the objective measure, second-period welfare for each consumer after the transaction is $-e^{\frac{r^2}{2}}$ and thus the transaction causes a reduction in the objectively expected utility of the two consumers from -1. Thus in this case, where arbitrage corrects mispricings rather than misallocations, welfare under the objective measure is very different. The arbitrage now causes an inefficient allocation of risk, reducing total social welfare and in particular harming the consumers. On top of this the consumers make a transfer to the the arbitrageur, which is neutral with respect to social welfare, but obviously harms the consumers.

2.3 Social value of arbitrage

Suppose one adopts as an objective-measure welfare criterion. Suppose there is an economy with a lot of arbitrage activity, some correcting misallocation and some correcting mispricings. What fraction of the value of arbitrage would have to be of the beneficial type in order for arbitrage to be on average social welfare enhancing? If we measure the value of arbitrage activity by the profits it generates for the arbitrageurs, then this value is proportional to the subjective welfare gain associated with the arbitrage. When arbitrage is beneficial, every dollar of this subjective welfare gain translates into a dollar of objective welfare gain. When arbitrage is harmful, $1-e^{-\frac{r^2}{2}}$ dollars of subjective welfare gain translates into $e^{\frac{r^2}{2}}-1$ dollars of objective welfare loss. Note that $e^{\frac{r^2}{2}}-1>1-e^{-\frac{r^2}{2}}$ by convexity of the exponential function. Intuitively, because the consumer is risk averse (and her utility therefore concave), her subjective utility gain under the arbitrage from her zero-risk starting position is smaller than the objective utility loss she suffers from the increase in risk. Also note that the greater r (the consumers' risk aversion) and the larger the stakes, the more this holds. However, this does not seem like a particularly general property.

To deal with the question of how many units of objective welfare loss accompany one unit of subjective welfare gain, I will now just assume this is given by some constant $\gamma \in \mathbb{R}_{++}$, though the above analysis suggests $\gamma > 1$ may be a plausible case to consider. In this case, the social value of one dollar of beneficial arbitrage is one dollar, where the social harm of one unit of socially harmful arbitrage is γ dollars. Suppose that a fraction β of all arbitrages are beneficial and the rest are harmful. Then arbitrage will on average be beneficial if $\beta > \frac{\gamma}{1+\gamma}$.

3 Discussion

The primary result above is that arbitrage that corrects misallocations of risk across segmented markets is beneficial, while arbitrage that eliminates mispricings is socially harmful. Financial economists, in the literature on the limits of arbitrage initiated by DeLong et al. (1990) and Shleifer and Vishny (1997), have spent a lot of time trying to understand when arbitrageurs will and will not be able to restore informational efficiency to pricing. An implicit assumption in this analysis is that efficient pricing is normatively relevant and probably normatively valuable. This model shows that this result is neither necessary nor even intuitive. In about the simplest model of arbitrage one could write down, correcting for the mistaken beliefs of behavioral traders is harmful, at least under the objective welfare measure.

There are three main problem in interpreting these results. First, how should one understand market segmentation in this context? What prevents trade and leads to arbitrage opportunities? Second, does one need to take "subjective mistaken beliefs" seriously in order to believe the narrative above? Or might there be more classical distortions that give rise to the harmful arbitrage argument? Finally, are there reasons, other than the imposition of an "objective" and therefore somewhat paternalistic welfare function that one might be concerned about the objective misallocation of risk as a result of harmful arbitrage?

3.1 Market segmentation

Clearly, some form of segmentation of markets is necessary to allow arbitrageurs to make profits. This seemingly ad hoc assumption of segmentation is therefore crucial to my model. However, it can be interpreted in a few, fairly intuitive ways:

1. Physical separation: The simplest and most straightforward interpretation is to consider two physically separated markets. The spice trade from India to the West would be a classic case. In modern finance, the segmented markets for Royal Dutch and Shell provide a rough example. A very clean example is crossed markets across exchanges.

2. Market incompleteness: A market for certain types of risk may not exist in the absence of the entrance of an market maker or financial innovator. A classic example would be the introduction of tradeable housing futures indices.

3. Mathematical and informational arbitrage: Two assets (or position) may be identical (or near identical), but may not be known to be so. Many quantitative arbitrage strategies are based on identifying such opportunities. The classic example would be put-call parity. In this view an arbitrageur represents a quantitative hedge fund that discovers such mathematical arbitrage opportunities and thereby connects the segmented markets for the two (mathematically equivalent) sides of a given trade. If one allows Bayesian uncertainty about logic statements (i.e. "Does $2+2=4$?") then these arbitrageurs can be viewed has having information others do not. In fact, any sort of information (not necessarily mathematical) that makes two assets equivalent to one another (say news of a merger) creates an opportunity for arbitrage of the segmented markets variety I model. I suspect that the entrance of other types of informed traders may be harmful as well in markets where investors act against their own interests.

4. Temporal (expectational) distance: My argument may also be understood within the contexts of a more classical model of arbitrage across time. For example in the classic DeLong et al. (1990) model of arbitrage, "market segmentation" is across time. Prices are (rationally) expected to rise or fall across time, but this differential can only be exploited by arbitrageurs who understand this. Thus in their model the "behavioral" confusion of traders generates both of the crucial elements of the harmful arbitrage model[6] : market segmentation (across time)

[6]Note that in their model, arbitrage smooths price volatility and therefore reduces risk to short-lived traders, a dynamic element abstracted away from by my simple two-period setting. However, given that market under-reaction as well as over-reaction has been observed and the existence of fundamental risk in most assets, it seems

and investor actions against their own best interest. However, one could imagine temporal segmentation that led to beneficial arbitrage: rather than behavioral traders, there might be long-term-oriented but rational "buy-and-hold" investors who are unwilling or unable to engage in the speculative purchase (or short-sale) and resale of an asset. If an asset is in over-supplied (for some reason) today relative to tomorrow, there might be cross-temporal beneficial arbitrage. Nonetheless this story is somewhat strained; it seems unlikely that investors would lack the foresight to make cross-temporal trades without also being confused about the values of assets at a particular time. Thus if such arbitrage across time is important then one might believe that harmful arbitrage may be more common than beneficial arbitrage.

3.2 Mistaken subjective beliefs

Many economists are uncomfortable with making welfare judgements in markets where traders have mistaken beliefs; some object to the very notion that certain beliefs should be viewed as "mistaken". While my assumption about the existence of confused market participants largely drives my analysis of harmful arbitrage, this assumption need not be taken too literally. While I consider confused investors to be an important reason for the relevance of the argument above, my assumption could also be viewed as a reduced form for other distortions in the market.

To take an example particularly relevant given recent events, suppose an individual hires a firm to manage her portfolio. Suppose that the individual can observe the expected payoff of the asset, but cannot directly observe its risk characteristics, except for some rating given to the instrument by a credit rating agency. A credit rating agency's judgement may be imperfect or corrupt and individuals may be uncertain about the covariance of relevant risk characteristics and expected payoffs of assets. Then an asset manager informed about the risk characteristics will have a natural incentive to seek out the most high yielding assets in a given rating class. Arbitrage opportunities that allow such a money manager access to a high risk, slightly higher return, well-rated asset will then harm the manager's clients. Thus "mistaken beliefs" type behavior by individuals may be driven by regulatory and agency problems, not simply pure behavioral confusion. This story

that generally the risk-generating potential of arbitrage will dominated its price smoothing effects in this harmful arbitrage setting. Hong and Stein (1999) provide a brief survey of evidence for (short-term) under-reaction and (long-term) over-reaction.

seems reasonable in light of recent reports of mutual funds and banks holding triple A rated, but highly complex, positions in Collateralized Debt Obligations widely believed to be far riskier than their ratings suggest. More generally, if investors cannot perfectly monitor fund managers, it seems plausible that in many settings the liquidity provided by arbitrage can harmfully expand the choice set of the fund manager. Clearly this argument should be developed more fully[7], but the basic point is that asymmetric information and perhaps other market distortions (not merely imperfect rationality) can lead to harmful arbitrage.

3.3 Objective welfare criterion

While some, such as Campbell (2006), accept and have argued for an objective-measure welfare criterion, even economists who accept the importance of noise traders reject are cautious about normative evaluation choices made by confused investors. While this may be less problematic under the interpretation, stressed above, of my model as a reduced form for agency problems or other non-behavioral distortions, there are undoubtedly many economists who will find the objective welfare criterion unpersuasive. However, it is important to note that even from this perspective, the model above obviously has the (psuedo-positive) implication that arbitrage may not create a more "efficient" allocation of risk in the sense that economists typically use the word. That is, arbitrage may lead to greater concentrations of undiversified risk, rather than greater diversification. This point is relevant regardless of whether one associate this "efficiency" with an improvement in welfare or not. Recent ructions in the financial markets have involved many financial organizations coming to the realization that they bore risks they did not fully understand. A primary driving factor in their coming to hold this risk was the "liquidity" provided in markets for complex derivatives by hedge funds and financial innovators. While the standard economic narrative (along the lines of the beneficial arbitrage model above) states that financial innovation and arbitrage will spread risk evenly over the population, allocating it to those best able to bear it, the model here suggests that, at least in some cases, arbitrage may help allocate risk to those least able to understand what they are getting into.

If there are negative externalities associated with individuals and institutions holding undiversi-

[7] I am working on a project in this direction in collaboration with Josh Schwartzstein.

fied portfolios, then the model may have normative implications even if one rejects a purely objective welfare criterion. Not all individual risk is really individual; banks that cannot perfectly monitor their borrowers, governments that find it hard to bail out hard-hit individuals may absorb some of the negative consequences of undiversified individual portfolios and family members uninvolved in financial decision making all bear the burden of poor financial decisions. Thing are worse in the case of financial institutions, where undiversified portfolios can have wide-ranging implications for the economy, not fully balanced by potential profits their ruin brings to those on the other side of their trades. Economists care about efficient allocations of risk not simply because of the welfare their bring to the individuals involved, but the macroeconomic stability they underpin. Furthermore, individuals who do not entirely understand the risk characteristics of the products they hold may be more likely to panic or otherwise act in a socially inefficient manner[8] when loses are realized than the objectively efficient bearers of that risk. Again, the recent financial instability undermines the argument that inefficient allocations of risk are alright, so long as individuals choose them.

4 An extension with capital allocation

Returning to the simpler, objective welfare criterion, there are two simple directions in which the model can naturally be extended. First, my model ignores the fact that asset prices often play a role in the allocation of capital (ie in real economic decisions) not just in allocating risk. In this section I develop a simple model of this situation, arguing that if a particular asset price is important for the allocation of capital or otherwise as a signal for real decisions, then arbitrage that leads to more informationally efficient prices is less likely to be very harmful, but also less likely to be strongly beneficial. Intuitively, elastic capital substitution both causes behavioral mispricings to distort real decisions and alleviates the harm caused by misallocations of risk. Increases in the costs of substitution (holding constant the equilibrium quantity) will always make arbitrage more

[8]One might suspect that "confused consumers" may hold two very different models of the asset in their mind. They may assume that the probability of model 1, under which the asset has very little risk (say the risk is lower bounded), is much higher and invest in the asset on the basis of this model. If this model is in fact incorrect, it may be falsified very rapidly, leading these confused investors to quickly unwind their positions causing market instability. On the other hand investors who understand the distribution of returns may be unlikely to respond in such a "discontinuous" way to unfavorable information and therefore disinvest in a poorly performing asset more smoothly. I hope to explore this argument further in a later paper.

beneficial, however. Less ambiguously, I treated profits by arbitrageurs as a pure transfer from consumers to arbitrageurs. In reality, much of the profits made by arbitrageurs will be competed away in efforts to discover arbitrage opportunities in the first place. In the next section, I will consider another simple general equilibrium model intended to deal with this point; there I will argue that, because arbitrage is costly and consumes productive labor resources may be more of a concern than the simple model suggests.

For a very simple analysis of the allocation of capital (real resources) that responds to asset prices, I augment the simple model by assuming that there exists a technology available to each (representative) consumer through which the asset may be produced or disposed of at a cost of $\frac{c(\rho,k)|z|^\rho}{\rho}$ where z is the net amount produced $\rho > 1$ and

$$c(\rho, k) \equiv k^{1-\rho}(1-k)r^2 e^{\frac{r^2(1-k)^2}{2}}$$

This technology may be interpreted as a real sector that receives funding by selling the asset; high (low) asset prices encourage (discourage) investment in this sector. Consider the outcome without arbitrage in both the beneficial arbitrage settings. Under beneficial arbitrage, obviously both consumers would choose to move at least somewhat towards 0 net supply of the asset, but not all of the way to this outcome, given the associated costs. Given the symmetry of the two representative consumer, I will consider only the case of the consumer with an initial endowment of -1 units of the asset. If z units of the asset are produced, its price (using the same calculations in the basic model) will be $(1-z)r^2 e^{\frac{r^2(1-z)^2}{2}}$. Optimal production of the asset the requires that

$$(1-z)r^2 e^{\frac{r^2(1-z)^2}{2}} = c(\rho, k) z^{\rho-1}$$

By convexity of the production function for the asset and concavity of utility, the equilibrium is uniquely defined by this expression. Clearly, $z = k$ satisfies this equality and therefore is the unique equilibrium. Thus k can be seen as a parameter determining how much of the asset is created in equilibrium and ρ measures the convexity of the cost function, holding constant the equilibrium asset production. Social welfare is given by the sum of expected utility of next period's consumption and the cost of producing the asset

$$W_1(\rho,k) \equiv -e^{\frac{r^2(1-k)^2}{2}} - \int_0^k c(\rho,k)z^{\rho-1}dz = -e^{\frac{r^2(1-k)^2}{2}} - \frac{c(\rho,k)k^\rho}{\rho} = -e^{\frac{r^2(1-k)^2}{2}} - \frac{k(1-k)r^2 e^{\frac{r^2(1-k)^2}{2}}}{\rho} =$$

$$-e^{\frac{r^2(1-k)^2}{2}}\left(\frac{\rho + k(1-k)r^2}{\rho}\right)$$

Note that W_1 is clearly increasing in ρ and can be shown[9] to be increasing in k. Thus the welfare under no arbitrage is increasing in the amount of substitution in the absence of arbitrage and the convexity of the costs substitution, neither of which is surprising.

When arbitrage occurs I again assume the most efficient allocation is reached: both consumers end up with 0 of the good and nothing is produced or disposed of. By symmetry, total welfare gain as a consequence of the arbitrage is $-\left[1 + W_1(\rho,k)\right]$, which is clearly decreasing in k and ρ. The more efficient and less costly is substitution, the less valuable arbitrage is in the beneficial arbitrage setting, as the gains to arbitrage can cheaply be achieved through producing or disposing of the asset. Intuitively this corresponds to the case where the initial misallocation is not strongly distortive because technology for the consumers choose to insure themselves cheaply. Intuitively, if the British can grow spices and the Indians manufacture luxury goods (albeit expensively), then the value of the spice trade is less than it would be in the absence of these substitutes.

Now consider the outcome without arbitrage in the harmful arbitrage setting. Again by symmetry, I focus on the market where investors mistakenly believe the asset is worth more than it truly is. If z of the asset is produced, calculations as above show that the price of the asset is $(1-z)r^2 e^{-\frac{r^2}{2}} e^{\frac{r^2(1-z)^2}{2}}$. Now let the cost of producing the asset be $\frac{e^{-\frac{r^2}{2}} c(\rho,k)|z|^\rho}{\rho}$. Note that the production function here is not exactly the same as the production function in the beneficial arbitrage

[9]Taking a derivative and doing some algebra

$$\frac{\partial W_1}{\partial k} = \frac{e^{\frac{r^2(1-k)^2}{2}} r^2}{\rho}\left(r^2 k[1-k]^2 + 2k + \rho[1-k] - 1\right)$$

The sign of this expression is clearly determined by the sign of

$$r^2 k[1-k]^2 + 2k + \rho(1-k) - 1 > 2k + \rho(1-k) - 1 > 2k + 1 - k - 1 = k > 0$$

context. However the adjustment is slight and essentially the two parameterizations represent the same substance: ρ, holding k constant, controls the cost of substitution, while k, holding ρ constant controls the quantity of substitution. Again, by construction, a quantity k of the good is produced in (arbitrage-free) equilibrium. Subjective welfare without arbitrage is

$$W_2(\rho,k) = -e^{-kr^2+\frac{k^2r^2}{2}} - e^{-\frac{r^2}{2}}c(\rho,k)\int_0^k z^\rho dz = -e^{-\frac{r^2}{2}}e^{\frac{r^2(1-k)^2}{2}}\left(\frac{\rho+k(1-k)r^2}{\rho}\right)$$

By the same reasoning as before, subjective welfare without arbitrage is increasing, and therefore subjective gains from from arbitrage are decreasing, in both ρ and k. Welfare under the objective measure is

$$W_2'(\rho,k) = -e^{\frac{k^2r^2}{2}} - e^{-\frac{r^2}{2}}c(\rho,k)\int_0^k z^\rho dz = -e^{-\frac{r^2}{2}}e^{\frac{r^2(1-k)^2}{2}}\left(\frac{\rho e^{kr^2}+k(1-k)r^2}{\rho}\right)$$

Just as before, clearly objective welfare is increasing in ρ (as substitution is less costly), but now it is decreasing in k (except for a few pathological parameter values[10]). Therefore the objective losses (or negative of gains) from arbitrage are increasing in ρ and (except for pathological cases) decreasing in k. The subjective welfare gain from arbitrage is $-\left[1+W_2(\rho,k)\right]$ and the objective welfare loss from arbitrage is $-\left[W_2'(\rho,k)+e^{-\frac{r^2}{2}}\right]$. Note that if we take the limit of the second expression as $k \nearrow 1$ and $\rho \searrow 1$ is 0, so[11] arbitrage remains harmful. This is obviously a special property of the model here, resulting from two properties:

1. My assumption of convex costs rules out very costly substitution.

[10]Taking the derivative with respect to k yields

$$\frac{r^2 e^{\frac{(k-2)kr^2}{2}}\left(rk\left[1-k\right]^2+k\left[2-e^{kr^2}\rho\right]\right)}{\rho}$$

the sign of which is clearly determined by the sign of

$$rk\left[1-k\right]^2+k\left[2-e^{kr^2}\rho\right]$$

Under either the two following conditions, this can be shown (by simple algebraic manipulations) to be negative:

1. $k > .39$ (computational simulations indicate that as ρ grows above 1 this bound can be substantially reduced; for $\rho = 2$, any value of $k > .1$ appears to suffice)
2. $r < 1$ (computational techniques can be used to prove that this continues to hold so long as $r < 4$)

[11]Subject to the caveats about the monotonicity in k and ρ.

2. As discussed in the simple model, the objective welfare losses are always large relative to subjective welfare gains. This is a consequence of my extremely simple model and likely does not generalize.

This section is a bit more complicated than the others, but a few simple messages can be taken away. One might think that arbitrage is valuable because it helps bring prices back to fundamentals, sending better signals for decisions in the real economy (the allocation of capital). Does taking into account this important role of arbitrage substantially reverse my conclusions about the potentially harmful effects of arbitrage? The possibility of such real effects (an increase in k above 0) does in fact lessen (though not necessarily reverse) the harmful effects of arbitrage in the harmful arbitrage setting. However, it also offsets the benefits of arbitrage in the beneficial arbitrage setting! A decrease in the convexity of substitution costs (an increase in infra-marginal substitution costs) holding constant the amount of substitution increases the benefits of arbitrage in both cases. Thus while admitting that asset prices may have real effects may (though need not) mitigate the potential for arbitrage to be harmful, it does not eliminate it.

5 An extension with costly arbitrage

In this section, I consider costly arbitrage that causes the rents from arbitrage to be dissipated by competition to discover arbitrage opportunities. Agents in the economy live on the unit interval. They are distributed uniformly on the interval and their measure is normalized to 1. Half of agents (independent of where they live) are type A agents, half type B. Thus in any infinitesimal interval dx on the unit interval $\frac{dx}{2}$ consumers of each type live. Consumer living at point x on the interval can only hold asset type x. Each point x therefore represents some (potential) arbitrage opportunity between two segmented markets (A and B) at x. All consumers have the same utility function as in the basic model and as in the model there are two periods. With probability β, independently and identically for each x, consumers at x are in the beneficial arbitrage situation and with probability $1 - \beta$ they are in the harmful arbitrage situation. All beneficial and harmful arbitrages lead to a subjective welfare gain of 1, α of which is captured by the arbitrageur (if an arbitrage occurs at x). Harmful arbitrage generates an objective welfare harm of γ.

In addition to asset markets, there are two productive sectors in the economy. One produces at constant returns to scale (with a return of 1), using labor as its only input. The second sector is the arbitrage-producing sector. If it hires l laborers it is able to exploit all arbitrage opportunities on the line between 0 and l^{δ} where $\delta < 1$. The profits of the arbitrage sector are divided evenly among the population. Both sectors of the economy operate in a common competitive labor markets. A social planner may charge ad-valorem tax at rate τ on the income of the arbitrage sector and the proceeds are distributed lump sum to the population. I am, of course, not suggesting that such a tax is a desirable policy tool, but rather use it as a simple means of measuring the harms or benefits (externalities) associated with arbitrage on the margin.

Firm optimality requires that wages in the arbitrage sector are $\frac{(1-\tau)\alpha\delta}{l^{1-\delta}}$ which must equal 1 at equilibrium. Thus at equilibrium a fraction $\big([1-\tau]\alpha\delta\big)^{\frac{1}{1-\delta}}$ of workers are employed in the arbitrage sector and so a fraction $\big([1-\tau]\alpha\delta\big)^{\frac{\delta}{1-\delta}}$ of all arbitrage opportunities are exploited. Social welfare (assuming equal weighting of all individuals' utilities) under the objective measure is just total product plus the objective gains (or losses) from the arbitrage by consumers. Note that with every arbitrage, consumers gain (if negative lose):

$$(1-\alpha)\beta - (\alpha+\gamma)(1-\beta) = \beta - \alpha - \gamma(1-\beta)$$

Thus social welfare is

$$1 - \big([1-\tau]\alpha\delta\big)^{\frac{1}{1-\delta}} + \big[\alpha + \beta - \alpha - \gamma(1-\beta)\big]\big([1-\tau]\alpha\delta\big)^{\frac{\delta}{1-\delta}} = \big(\beta - \gamma[1-\beta] - [1-\tau]\alpha\delta\big)\big([1-\tau]\alpha\delta\big)^{\frac{\delta}{1-\delta}}$$

Optimality requires that

$$\alpha\delta\big([1-\tau]\alpha\delta\big)^{\frac{\delta}{1-\delta}} = \frac{\alpha\delta^2\big(\beta - \gamma[1-\beta] - [1-\tau]\alpha\delta\big)}{1-\delta}\big([1-\tau]\alpha\delta\big)^{\frac{2\delta-1}{1-\delta}}$$

$$[1-\tau]\alpha\delta = \frac{\delta\big(\beta - \gamma[1-\beta] - [1-\tau]\alpha\delta\big)}{1-\delta}$$

$$\alpha\delta[1-\tau]\left(1+\frac{\delta}{1-\delta}\right)=\frac{\delta\big(\beta-\gamma[1-\beta]\big)}{1-\delta}$$

$$1-\tau=\frac{\beta-\gamma(1-\beta)}{\alpha}$$

$$\tau^{\star}=\frac{\alpha+\gamma(1-\beta)-\beta}{\alpha}$$

Which is intuitive Ramsey pricing: $\alpha+\gamma(1-\beta)-\beta$ is the per-unit-output negative externality of arbitrage. If $\beta>\alpha+\gamma(1-\beta)$ then arbitrage has a positive (pecuniary or market-making) externality and thus should be subsidized. If $\gamma(1-\beta)\geq\beta$ (the condition for arbitrage to be harmful in the basic model) then arbitrage should be taxed at least 100% (ie prohibited) which is the same as in the simple model. However, taking into account the dissipation of arbitrage profits in the work of producing arbitrage, arbitrage has a negative externality if

$$\frac{\alpha+\gamma}{1+\gamma}>\beta$$

which is another simple formula for thinking about when arbitrage may be harmful (generate negative externalities) or at least not very beneficial (generate few positive externalities). The point is that if β is less than or at least not much greater than $\frac{\alpha+\gamma}{1+\gamma}$ then the arbitrage sector may inefficiently draw talent from professions like engineering and science that we think generate more positive externalities. Given the extraordinary fraction of talented individuals working in the finance (and arbitrage in particular) sector in recent years, this point seems particularly germane. For example, Kaplan and Rauh (2006) find that more than twice as many earners at various measures of the top end of the income spectrum are financial as non-financial executives. More than a third of graduating undergraduate classes at Princeton in recent years have taken jobs on Wall Street[12]. Given the particularly strong draw of arbitrage and financially innovative activities for individuals (those trained in science and engineering[13]) most likely to otherwise enter careers with high positive

[12] FIND CITATION.
[13] CITE.

externalities, this trend may a cause for concern.

6 Conclusion

Economists often associate informational efficiency with allocative efficiency. Arbitrage and financial innovation that increase the first, it is often argued, help smooth risk over the population, as well as promoting the efficient allocation of capital. In this paper, I challenge this argument. When confused consumers or other distortions are the source of opportunities, arbitrage and market making may be socially harmful. While the role of arbitrage in ensuring an efficient allocation of capital may weigh in favor of arbitrage, the fact that arbitrage is costly to produce weighs against it.

While my argument has some implications for interpreting recent events in financial markets and understanding the consequences of the influx of talented workers into finance, it raises many more questions than it answers. Which distortions can give rise to harmful arbitrage opportunities? Can we measure how common harmful arbitrage is relative to beneficial arbitrage? Is there something particularly harmful to financial stability about risk being allocated to investors who "do not know what they are getting themselves into"? What, if any, are appropriate regulatory responses to harmful arbitrage[14] or the distortions that give rise to it? I hope to address some aspects of these questions in future research.

[14] For example, might securities regulators consider forcing "innovative" financial products (such as Collateralized Debt Obligations) to go through a screening process similar to what new medicines routinely experience to determine whether they have a legitimate economic (risk-allocation) purpose?

References

Baker, M., J. C. Stein and J. Wurgler (2002), When does the market matter? stock prices and the investment of equity-dependent firms, Technical report, National Bureau of Economic Research.

Campbell, J. Y. (2006), 'Household finance', *Journal of Finance* **61**(4), 1553–1604.

DeLong, J. B., L. H. Summers, A. Shleifer and R. Waldmann (1990), 'Noise trader risk in financial markets', *Journal of Political Economy* **98**, 703–738.

Hong, H. and J. C. Stein (1999), 'A unified theory of underreaction, momentum trading and overreaction in asset markets', **54**(6).

Kaplan, S. and J. Rauh (2006), Wall street and main street: What contributes to the rise in the highest incomes?, Technical report, University of Chicago.

Murphy, K. M., A. Shleifer and R. W. Vishny (1991), 'Allocation of talent: Implications for growth', *Quarterly Journal of Economics* **106**(2), 503–530.

Shiller, R. J. (1993), *Marco Markets: Creating Institutions for Managing Society's Largest Economic Risks*.

Shleifer, A. and R. W. Vishny (1997), 'The limits of arbitrage', *Journal of Finance* **52**(1), 35–55.

The Economist (2007), 'Securitisation: When it goes wrong...', (September 20th).